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Second-harmonic generation excited by a rotating Laguerre-Gaussian beam

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Experimental data demonstrate that unlike linear optical processes, an optical Laguerre-Gaussian beam of frequency ω, with topological charge m, rotating with angular frequency $\Omega \ll \omega$, may not be considered as a monochromatic beam with the shifted frequency $\omega + m\Omega$ (Doppler angular shift) for the second-harmonic generation nonlinear process.

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Propagation of optical beams with orbital angular momentum originating from the azimuthal phase dependence of the field distribution has been actively studied during the last twenty years [1]. A special interest in this case has been light-matter interactions. Previously, the interaction of these beams with atoms was studied theoretically [2] and experimentally [3]. Second-harmonic generation (SHG) excited by Laguerre-Gaussian (LG) beams, a specific type of beam with orbital angular momentum, was first observed for a low transformation efficiency [4,5] and for strong coupling between the waves of fundamental and double frequencies [6,7].

Analogous to a circularly polarized wave transmitted through a rotating half-wave plate, it has been proposed that the rotation of such beams with an angular velocity much less than the optical frequency will impart a frequency shift [1]. Unlike a Gaussian beam, the phase front of a helical beam is not invariant under the rotation. The rotation of such a beam is indistinguishable from its temporal evolution: a light beam of frequency ω with an $e^{im\phi}$ phase structure and m (topological charge) intertwined helical wave fronts (ϕ is the angular coordinate), rotating with angular frequency Ω, has the shifted frequency $\omega + m\Omega$ (Doppler angular shift).

A question arises, however, as to whether this shift is a real shift of the beam frequency in the sense that the rotating beam with a helicoidal wave front is considered to be a monochromatic beam of constant amplitude with frequency $\omega + m\Omega$. The first experimental study of this problem in the microwave frequency range was performed in [8]. The rotationally induced frequency shift was measured by dividing an LG beam in half with a value of m and recording the interference between one transmitted through a rotating Dove prism and the other transmitted through a nonrotating Dove prism. The two beams were then focused to a photodetector and the output signal was recorded as the Dove prism was rotated. By rotation of the prism with a given frequency Ω, the signal changed periodically with a frequency $m\Omega$, and therefore, the presence of the Doppler angular shift in the rotating LG beams was demonstrated.

In this work, we present experimental results of a similar experiment but in a nonlinear case, in particular, for the SHG with rotating LG beams. We suppose that an input LG beam of fundamental frequency ω, with an $e^{im\phi}$ phase structure, rotating with an angular frequency $\Omega \ll \omega$, enters into a nonlinear optical medium. A question that we will answer in this work is: what is the angular rotating frequency of the beam of double frequency that is generated due to the nonlinear second order process? To our knowledge, there is no previous theoretical work that describes the use of rotating LG beams for SHG, therefore we believe our experimental results may initialize theoretical studies of the nonlinear optical processes with these types of optical beams.

The experimental setup is shown in Fig. 1. The light source was a 1064-nm, Q-switched, mode-locked Nd:YAG laser providing 35-ps pulses with a 25-Hz repetition rate. A beam splitter (BS_1) splits an expanded linearly polarized beam of 8 mm in diameter to two channels.

In the reference channel, the beam of fundamental frequency propagates coaxially with the beam of double frequency generated by a potassium titanyl phosphate crystal C_1. A lens (L_1) focuses the incident beam into the crystal in order to achieve the beam power density that is sufficient to generate the SHG beam. A lens (L_2) collimates both the beam of fundamental frequency and the beam of double frequency. A delay line (DL) in the reference channel permits one to equalize the optical paths of the beams both in the signal and reference channels. The length of the crystal (C_1) is short so that the phase matching condition is satisfied for all plane waves that form the converging incident beam.

In the signal channel, a rotating optical beam can be produced by first generating a beam with helical wave front with a phase mask and then rotating this beam using a π convertor consisting of cylindrical lenses [9] or a Dove prism [5]. We use a rotating spiral zone plate RSZP, because in this case it is much easier to circumvent difficulties caused by translated or angular deviation of rotating lenses and Dove prisms. The spiral zone plates [10] have a diameter of 10 mm, which is greater then the incident beam size to prevent edge diffraction distortions. The first diffraction order is focused by the spiral zone plate on the optical axis. A pinhole (P) at the focal plane of the mask selects the diffraction order that is then collimated by another lens (L_3). This collimated beam passes through a second nonlinear crystal (C_2) resulting in a beam of double frequency. The SHG process with a weak coupling between the beams of both frequencies was used. After a beam splitter (BS_2), the output beams of the fundamental and double frequencies generated by the crystal C_2 overlap with the beams of both frequencies propagating in the reference channel

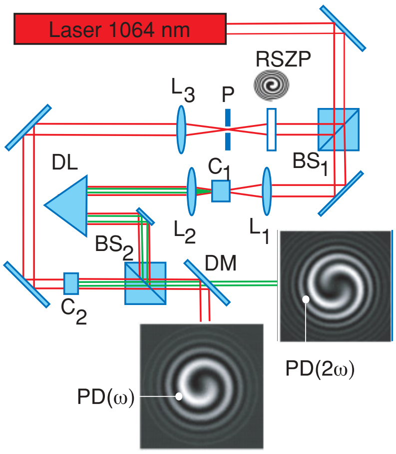

FIG. 1. (Color online) Experimental setup. The symbols used for different elements are explained in the main text. White circles at the interference patterns show the aperture of the photodetectors.

producing spiral interference patterns PD(ω) and PD(2ω). A dichroic mirror (DM) and bandpass optical filters permit us to spatially split the interference patterns. As the spiral zone plate RSZP is rotated, the interference patterns of both frequencies rotate. The rotation rate of the spiral zone plate was 100 rpm, which produced a frequency of angular movement of about 1.6 Hz.

Figure 1 illustrates the principle of the measurement of the angular frequency of rotation of the interference patterns. Considering first the channel with the fundamental frequency, the photodetector PD(ω) has an aperture (shown as a white circle in the image of the interference pattern) much less than the distance between two neighboring bright lines of the spiral interference pattern. The photodetector generates a periodic signal as soon as the spiral zone plate RSZP in the signal channel rotates, providing the rotation of the interference fringes. The signal from the photodiode was analyzed by a digital oscilloscope that also permits calculation of the fast Fourier transform (FFT) spectra in real time. Spectra shown in the following figures were obtained using data acquired during a time interval of 20 s.

We first consider the results obtained when the beam of fundamental frequency in the signal channel propagates through the rotating zone plate with a single spiral (Fig. 2) which corresponds to the rotating LG beam with topological charge $m = 1$. As seen for the beam of fundamental frequency [Fig. 2(a)], the interference pattern is a single spiral. The Fourier spectrum of the photodetector signal when the zone plate rotates reveals a peak at 1.6 Hz, i.e., at the frequency of rotation of the zone plate. Hence, this beam rotates with the same angular velocity as the rotating mask. This test of the experimental scheme confirms previous results [8].

The interference pattern of the second-harmonic beam [Fig. 2(b)] is a double spiral due to the doubling of the azimuthal index [4]. The Fourier spectrum [Fig. 2(b)] shows a peak at 3.2 Hz, i.e., double the rotation frequency of the zone plate. However, the interference pattern is a double spiral, therefore we conclude that the second-harmonic beam rotates with the *same* angular frequency 1.6 Hz as the fundamental beam.

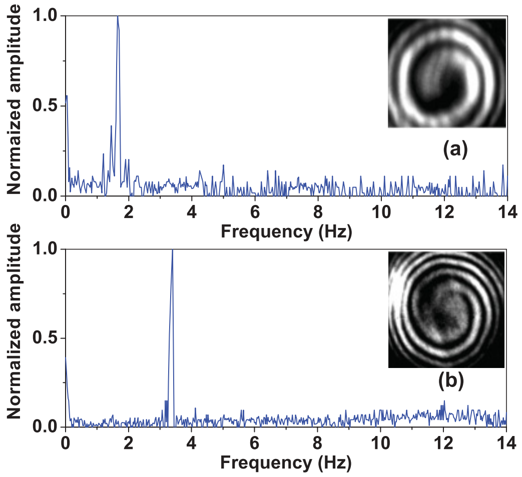

FIG. 2. (Color online) FFT spectra of the signals obtained from the photodetectors PD(ω) (a) and PD(2ω) (b). The insets are images of the corresponding interference patterns. The topological charge of the fundamental beam is $m = 1$.

Let us now consider the experiments when the fundamental beam carries a helical wave front with topological charge $m = 2$ (Fig. 3). To generate this beam, we use the double-spiral zone plate. The beating frequency of the fundamental beam is double the rotation frequency of the zone plate which again corresponds to [8]. However, the beating frequency of the second-harmonic beam is four times the rotation frequency of the mask. Understanding that the interference pattern is a fourfold spiral, we again conclude that the second-harmonic beam rotates with the *same* angular frequency (1.6 Hz) as the fundamental beam.

Therefore, by the SHG of the rotating LG beams, the second-harmonic beam rotates with the same angular velocity as the fundamental incident beam, and the nonlinear process does not produce the doubling of the rotation frequency of the incident beam of fundamental frequency.

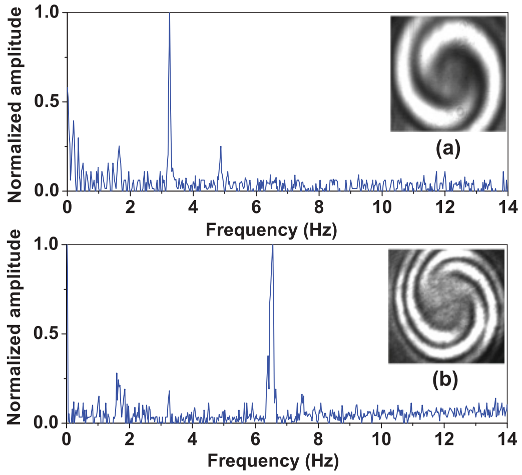

FIG. 3. (Color online) Spectra analogous to those presented in Fig. 2, but the topological charge of the fundamental beam is $m = 2$. The insets are images of the corresponding interference patterns.

On the basis of this, we conclude that unlike linear optical processes for the second-order nonlinear process, the optical LG beam of frequency ω, rotating with angular frequency $\Omega \ll \omega$, may not be considered as a monochromatic beam with shifted frequency $\omega + m\Omega$. An additional theoretical study is needed to explain this experimental fact.

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