Vortex streets in walking parametric wave mixing

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The combined effects of diffraction and Poynting vector walk-off in second-harmonic generation with pump beams that contain screw phase dislocations is addressed for what is believed to be the first time. We predict the spontaneous nucleation of multiple vortex twins whose subsequent explosion can yield quasi-aligned patterns of single-charge vortices.

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Singular light beams that contain topological wave-front dislocations are ubiquitous entities that display fascinating properties with widespread, important applications.[1,2] Screw dislocations, or vortices, are a common dislocation type. They are spiral phase ramps around a singularity in which the phase of the wave is undefined, and its amplitude vanishes. The order of the screw dislocation multiplied by its sign is referred to as the winding number, or the topological charge of the dislocation.

Vortices appear spontaneously in several settings, including in speckle fields, in optical cavities, and in doughnut laser modes, and they can be generated with phase masks or with astigmatic optical components. Vortices also form by self-wave-front modulation in nonlinear optical media.[3] In this context, parametric wave mixing of multiple waves propagating in quadratic nonlinear media is a fascinating possibility. Charge doubling in second-harmonic (SH) generation schemes[4,5] and sum- and difference-charge arithmetic operations in three-wave-mixing processes[6] have been demonstrated with moderate light intensities and wide pump beams. Spontaneous vortex-pair nucleation in seeded upconversion schemes[7] and vortex excitation in parametric amplification from quantum noise[8] constitute two additional examples of the phenomena that were discovered recently. Our goal in this Letter is to show that the combined effects of diffraction and Poynting vector walk-off open the door to a new range of possibilities. In particular, we predict the spontaneous nucleation of multiple vortex twins whose subsequent explosion under appropriate conditions yields quasi-aligned patterns of single-charge vortices.

We consider cw light propagation in a bulk quadratic nonlinear crystal under conditions for type I SH generation. The evolution of the slowly varying envelopes of the light beams is described by

$$i\,\frac{\partial a_1}{\partial \xi} - \frac{\alpha_1}{2}\,\nabla_\perp{}^2 a_1 + a_1{}^* a_2 \exp(-i\beta\xi) = 0\,, \quad (1)$$

$$i\,\frac{\partial a_2}{\partial \xi} - \frac{\alpha_2}{2}\,\nabla_\perp{}^2 a_2 - i\delta\,\frac{\partial a_2}{\partial x} + a_1{}^2 \exp(i\beta\xi) = 0\,, \quad (2)$$

where a_1 and a_2 are the normalized amplitudes of the fundamental frequency (FF) and the SH waves, respectively, $\alpha_1 = -1$, and $\alpha_2 = -k_1/k_2 \simeq -0.5$. Here k_ν, with $\nu = 1, 2$, are the wave numbers. The transverse coordinates are given in units of the beam width η, and the scaled propagation coordinate is $\xi = z/2l_{d1}$, with $l_{d1} = k_1\eta^2/2$. The parameter β is given by $\beta = k_1\eta^2\Delta k$, where $\Delta k = 2k_1 - k_2$ is the wave-vector mismatch. The parameter δ stands for the Poynting vector walk-off. It is given by $\delta = \pm 2l_{d1}/l_w$, where the walk-off length $l_w = \eta/|\rho|$; ρ is the walk-off angle.

We consider upconversion geometries with material and light conditions that yield negligible depletion of the pump FF beam. Then, a_1 can be assumed to be unaffected by the wave interaction, and hence the general solution of Eq. (2) can be formally written as

$$a_2(x, y, \xi) = \int_{-\infty}^{\infty} \mathrm{d}x' \int_{-\infty}^{\infty} \mathrm{d}y' a_2(x', y', \xi = 0) \times K(x - x', y - y', \xi) + \int_{-\infty}^{\infty} \mathrm{d}x' \int_{-\infty}^{\infty} \mathrm{d}y' \times \int_0^{\xi} \mathrm{d}\xi' a_1{}^2(x', y', \xi') K(x - x', y - y', \xi - \xi')\,. \quad (3)$$

Here $a_2(x, y, \xi = 0)$ stands for any SH seed that is present at the entrance face of the crystal, $a_1(x, y, \xi)$ is the linearly evolving FF input beam, and the Green function $K(u, v, w)$ is the walking free-space propagator at the SH frequency,

$$K(u, v, w) = \frac{i}{2\alpha_2\pi w} \exp\left[\frac{(u + \delta w)^2 + v^2}{i2\alpha_2 w}\right]\cdot \quad (4)$$

Here we consider unseeded geometries and input FF light beams constituted by Gaussian beams with nested vortices, with the general form[2]

$$a_1(x, y, \xi) = A_0\,\frac{[x + \mathrm{sign}(m) i y]^{|m|}}{(1 + i2\xi)^{|m|+1}} \exp\left(-\frac{x^2 + y^2}{1 + i2\xi}\right), \quad (5)$$

where m is the charge of the dislocations. Here we restrict ourselves to $m = 1$. In such a case, letting $\alpha_2 = -0.5$, substitution of Eq. (5) into Eq. (3) and integration over the transverse coordinates yield

$$a_2(x, y, \xi) = \frac{A_0{}^2}{(1 + i2\xi)^3} \int_0^{\xi} \frac{[x + \delta(\xi - \xi') + iy]^2}{1 + i2\xi'} \times \exp\left\{-2\,\frac{[x + \delta(\xi - \xi')]^2 + y^2}{1 + i2\xi}\right\} \exp(-i\beta\xi')\mathrm{d}\xi'. \quad (6)$$

When $\delta = 0$, the transverse and the integration coordinates are decoupled, and one obtains the known result for a SH beam with a double-charge vortex located at $(x, y) = (0, 0)$.[4–7] In the presence of Poynting vector walk-off, Eq. (6) can be evaluated numerically in a mesh of transverse points at each value of the propagation distance. We monitored the presence of screw dislocations in the SH beam visually by superposing the obtained SH beams with a reference-tilted plane wave. We then determined the number and the precise locations of all the dislocations by numerically harvesting all the complex zeroes of Eq. (6).

At short propagation distances and small walk-off parameters, diffraction and walk-off weakly affect the beam evolution. The only effect induced by the presence of walk-off is the splitting of the double-charge vortex into two single-charge vortices.[6] When $\xi \ll 1$, the SH beam Eq. (6) near the vortex core behaves as

$$a_2(x \ll 1, y \ll 1, \xi \ll 1) \approx A_0{}^2 \xi (x^2 - y^2 + x\delta\xi + \delta^2\xi^2/3 + i2xy + i\delta y\xi). \quad (7)$$

Thus, at short propagation distances the single-charge vortices are located at

$$x \approx -\frac{\delta\xi}{2}, \qquad y \approx \pm\frac{\delta\xi}{2\sqrt{3}}, \quad (8)$$

which coincides with the diffractionless result.[6] However, a new range of phenomena is discovered when both δ and the propagation distance are increased so that the competition between diffraction and walk-off can manifest itself. Figures 1 and 2 show representative examples of the situation that is encountered. The plots correspond to $\delta = 2$, which represents material and light conditions in which walk-off and diffraction compete on a similar footing, and to exact phase matching.

One finds that, as the SH beam propagates and walks off the location of the pump FF beam, multiple vortex twins that have a zero net topological charge are continuously nucleated. Such twins explode into the vortices with the positive and the negative charges that they contain,[9] which then move away from the birth point. Figure 1 displays the evolution of the transverse locations of all the single-charge vortices that are present in the SH beam as a function of propagation distance, as predicted by Eq. (6). Figures 2(a)–2(c) show the interferograms obtained at selected instances of the beam evolution. After the explosion of the nucleated vortex twins, the resulting single-charge vortices move away and interact with one another. Such interaction can include rotation, drift, and vortex–antivortex annihilation when the two meet at the same point, as is visible in Fig. 1(b). In any case, one remarkable feature revealed by the calculations is that the vortices that survive arrange themselves in a quasi-aligned geometry, in which all the vortices in each line have the same charge, either positive or negative. Figure 2(d) shows the amplitude of the SH beam that is observed under such conditions.

The behavior of the vortex pattern as a function of δ is complex. Figure 3 shows illustrative examples of the evolution. The plots show interferograms of the SH beam obtained at a fixed propagation distance for different values of δ. By and large, we found that increasing the walk-off parameter leads to nucleation of a larger number of vortex twins. Yet the propagation distance at which the twins are nucleated was observed to depend on the value of δ. One can conclude from the simulations that were performed and are illustrated in Fig. 3 that for a given crystal length there is an optimum value of δ for generating the largest number of twins. The value of the existing wavevector mismatch was found to affect the process of twin nucleation, explosion, and subsequent dynamics as well. Typical evolutions are shown in Fig. 4. A nonvanishing mismatch is found to increase the propagation distance required for twin nucleation, so that for a given crystal length, the larger the value of β, the

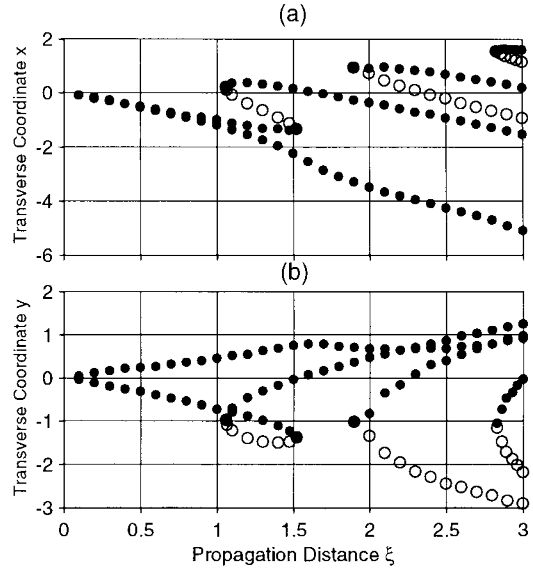

Fig. 1. Locations of all the single-charge vortices that are present in the SH beam as a function of crystal length. Filled circles, positive vortices such as those of the pump FF beam; open circles, negative vortices. $\delta = 2$, $\beta = 0$.

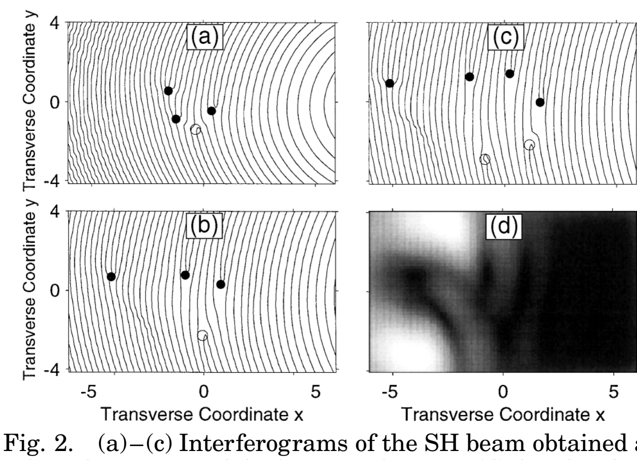

Fig. 2. (a)–(c) Interferograms of the SH beam obtained at selected instances of the beam evolution. (d) Amplitude of the SH beam shown in (b). $\delta = 2$, $\beta = 0$. Propagation distances: (a) $\xi = 1.2$, (b) $\xi = 2.4$, (c) $\xi = 3$.

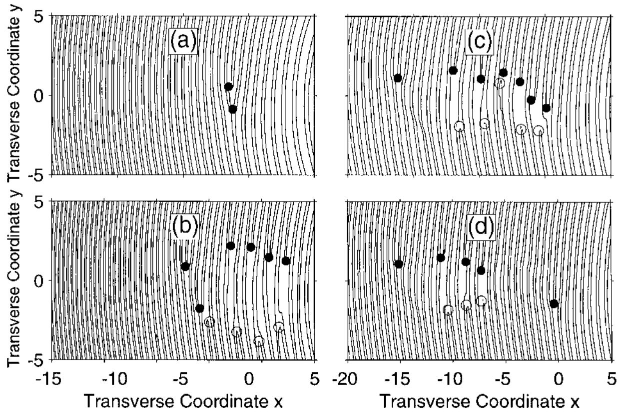

Fig. 3. Interferograms of the SH beam obtained at fixed crystal length for different walk-off strengths. $\xi = 4$, $\beta = 0$. Walk-off: (a) $\delta = 0.5$, (b) $\delta = 2$, (c) $\delta = 3$, (d) $\delta = 4$.

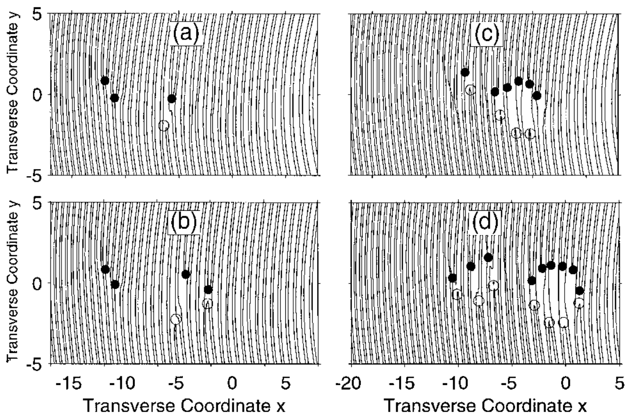

Fig. 4. Interferograms of the SH beam under conditions of nonvanishing wave-vector mismatch. Mismatch: (a), (b) $\beta = 3$; (c), (d) $\beta = -3$. Propagation distances: (a), (c) $\xi = 4$; (b), (d) $\xi = 5$. In all cases $\delta = 3$.

smaller the number of nucleated twins. This result is clearly visible in Figs. 4(a) and 4(c), which are to be compared with Fig. 3(c). One finds that for a given crystal length and δ no twins are nucleated beyond a critical value of β. As shown in Fig. 4, twin nucleation tends to be more difficult at positive mismatches than at $\beta < 0$. Disordered and complex vortex structures were sometimes observed at negative mismatch, but only for specific, narrow-range values of all ξ, δ, and β. The ordered structures shown in Fig. 4 illustrate the features that were observed in the large majority of cases.

Figures 1–4 were all obtained from the semianalytical solution Eq. (6) and backed up with numerical simulations of the full governing equations (1) and (2) performed with a split-step Fourier algorithm. Within the resolution that was achievable with split-step meshes, perfect agreement between the predictions of Eq. (6) and the numerical simulations was always obtained under conditions in which the negligible FF pump approximation held. Failure of such conditions led to a totally new situation in which there were dislocation exchanges between the FF and the SH beams.[10]

Eventually we notice the intuitive similarity between the phenomena discovered here and the formation of Bénard–von Karman vortex streets in viscous-fluid flow behind a fixed obstacle or the emission of vortex twins behind a moving obstacle in superflows.[11] However, there are crucial differences. For example, Bénard–von Karman vortex formation relies on the dynamics of the viscous boundary layer at the static obstacle, a mechanism that is absent in our case, in which the vortex twins are formed by the interference pattern dictated by Eq. (6). Yet the analogy existing between fluid dynamics and similar optical systems[12,13] might yield new, important insights into the present case that remain to be elaborated.

In conclusion, we have shown that under appropriate conditions the combined effects of diffraction and Poynting vector walk-off in parametric wave mixing with pump beams that contain screw phase dislocations induce spontaneous nucleation of multiple vortex twins whose subsequent explosion can yield quasi-aligned patterns of single-charge vortices.

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